

# Phase boundaries in vertically vibrated granular materials

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## Abstract

A boundary structure is observed in beds of vertically vibrated granular materials separating regions of opposite phase. This structure is investigated and it is identified as giving rise to both a global kink and hexagonal patterns. A global phase diagram of surface patterns on a vibrated bed is presented. © 1998 Published by Elsevier Science B.V.

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In recent years, there has been a dramatic surge in interest in the behavior of granular materials. Of particularly high profile has been the compelling work on vertical vibration and pattern formation. In beds of small grains, convection, heaping and surface patterning have been reported [1–3]. Among the most detailed investigations of the surface pattern characteristics of such systems is a set of reports on surface geometries categorizing the dynamics of shallow layers of grains [4–6]. In this Letter, we report on a transitional structure in the phase change between these geometrical patterns.

We examined the behavior of brass spheres of radius 0.15–0.18 mm in a bed of 20 grains in depth. The grains were vertically vibrated in a chamber consisting of a horizontal aluminum platform stabilized by vertical stainless steel shafts supporting a cylindrical lucite wall of radius 45 mm and height 50 mm. The chamber was vertically vibrated using a standard laboratory vibrator driven by an amplified signal generator. The chamber and driver were placed in a vacuum chamber and the pressure was reduced to below to minimize gas-related effects [2]. Surface patterns were observed and recorded using both high-speed digital

photography and strobed light video. The oscillation amplitude  $A$  was measured optically, from which the acceleration amplitude  $\Gamma = 4\pi^2 A f^2 g^{-1}$  was derived, where  $g$ , the acceleration of gravity is scaled to 1.

Our results conform to those reported by several in-

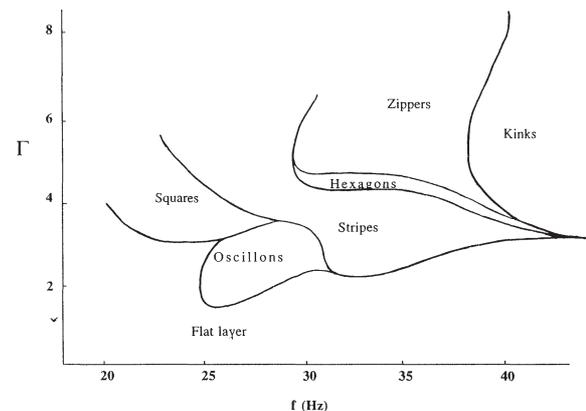


Fig. 1. Phase Diagram for 0.15–0.18 mm brass spheres of layer depth 20. Regions of square, flat and oscillon behavior are entirely bounded by stripes. The system is hysteretic and the region boundaries depicted were constructed from data taken with decreasing  $\Gamma$ . As the zipper structure's amplitude increases, hexagons bleb off before the pattern reverts to stripes.

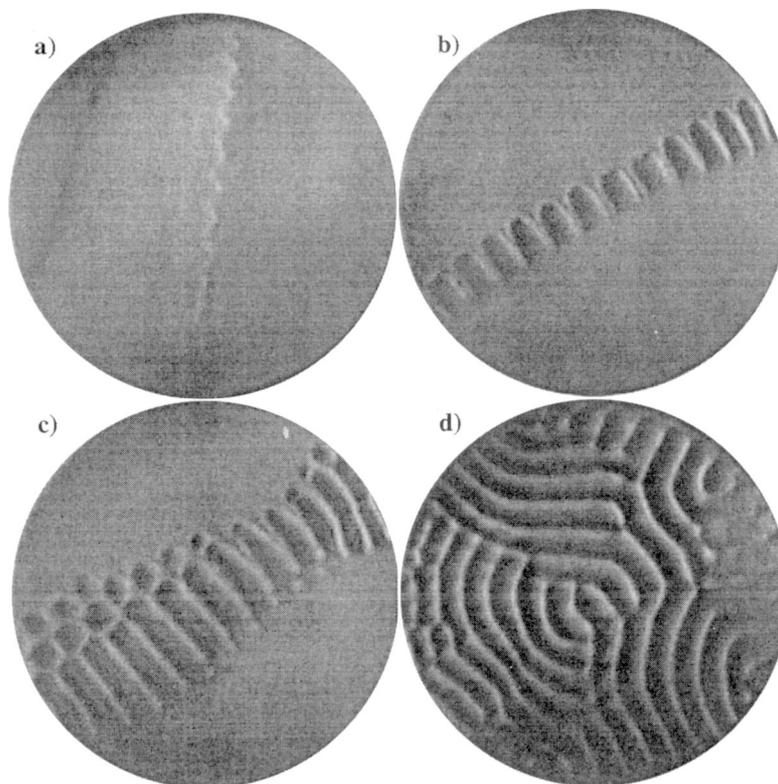


Fig. 2. The transition from global kink to stripes. (a) At  $f = 41.4$  Hz,  $\Gamma = 5.00$ , a kink separates regions of opposite phase. (b) At  $f = 33.5$  Hz,  $\Gamma = 5.04$ , the edge of the kink becomes scalloped with teeth entering from one region into the other forming a zipper. (c) At  $f = 34.6$  Hz,  $\Gamma = 4.69$ , the zipper opens further and hexagons begin to bleb off the ends of the teeth. (d) At  $f = 33.5$  Hz,  $\Gamma = 4.29$ , hexagons have given way to stripes.

investigators who observed square, stripe, hexagonal and kink surface patterns. However, it is noteworthy that in this experiment, the phase diagram differs somewhat from that presented elsewhere. Rather than observing a phase transition from squares to stripes (at higher frequency) and squares to hexagons (at higher  $\Gamma$ ), we observe the phase region of stripes to completely bound that of squares with increases in both frequency and  $\Gamma$  (Fig. 1). Our data mirror the results presented in Fig. 2 of Ref. [5] with regard to the phase region of oscillons. In addition, we note a distinction between the linear global boundary between out of phase regions (the “kink”) and an elongated, wavelike boundary which we call a “zipper”.

Of principal interest in our results are the complicated characteristics of a global boundary between regions of opposite phase. In the phase region explored here, the frequency of oscillation is high enough to

allow for period doubled motion. This permits the co-existence of oscillations with opposite phase. At sufficiently high frequency and amplitude, the bed resolves into large, phase-locked regions each dominating roughly half of the vessel. At high frequency, 38–45 Hz, and  $\Gamma = 2.6$ –8, this boundary consists of a very distinct and linear structure, the so-called “kink” (Fig. 2a).

However, below this region in frequency and/or  $\Gamma$  lies a scalloped pattern whose ridges become longer in a transition that resembles the opening of a zipper. As the zipper opens, the transverse teeth that jut out intrude on the oppositely phased region (Figs. 2b).

This pattern has been noted without comment [7], and very recently described as a subharmonic standing wave instability [6]. However, rather than an increase in wavelength with decrease in frequency as reported there, an increase in width (or amplitude) is

observed. As the teeth elongate, they eventually pinch off, forming hexagonal cells in the opposing phase region (Fig. 2c).

This continuing lengthening of the boundary results in the generation of closed hexagonal cells which bleb off from the teeth of the zipper. Finally, the global boundary disappears and a stripe pattern overwhelms the container (Fig. 2d). The lengthening of the boundary between regions of opposite phase, followed by the invasion of one region by cells of oppositely phased motion and a breakdown in the global boundary is central to the analysis of this system.

This progression is suggestive of a viscosity-like mechanism [8] in the fluidized layer that suppresses out-of-phase motion of neighboring grains. As the frequency and amplitude are increased, this effect becomes stronger resulting in absolute suppression of the out-of-phase mode in each of two competing regions. A global mounding phenomenon [9,10] on the plastic sublayer may explain the formation of the global kink and the lack of observations of complete dominance of the bed by a single phase.

The zipper structure is generic to a variety of materials observed under vibration including salt and spherical grains of various sizes. Further study is called for to precisely determine the dependence of total boundary length on vibrational energy. In addition, a model is needed that obtains from neighbor interaction a fre-

quency and amplitude dependent viscosity of the fluidized granular layer.

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