Velocity correlations in dense granular gases

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We report the statistical properties of spherical steel particles rolling on an inclined surface being driven by an oscillating wall. Strong dissipation occurs due to collisions between the particles and can be tuned by changing the number density. The velocities of the particles are observed to be correlated over large distances comparable to the system size. The distribution of velocities deviates strongly from a Gaussian. The degree of the deviation, as measured by the kurtosis of the distribution, is observed to be as much as four times the value corresponding to a Gaussian, signaling a significant breakdown of the assumption of negligible velocity correlations in a granular system.

From the study of landslides to the understanding of the formation of galaxies, systems of macroscopic particles are of fundamental interest to physicists and engineers [1,2]. To treat a granular system statistically, temperature must be replaced by a more suitable quantity. Granular temperature has gained considerable attention over the past two decades as a useful quantity for describing the properties of rapid granular flow [3–5]. If the collisions between particles are elastic, then we would expect on very general grounds that the velocity distribution is Gaussian and the velocity components are uncorrelated. However, dissipation due to inelasticity and friction can introduce correlations that make these properties of the velocity distribution suspect [6]. For example, a simulation of a system of inelastic particles yields long-range velocity correlations [7].

The first experiments with steel particles excited by a vertically vibrated container indicated that the velocity distribution is Gaussian [8]. Using advances in high-speed digital imaging, later experiments have shown deviations in the tails of the distribution from a Gaussian [9–11]. It also has been claimed that the distribution of the velocity component, v_x, in the direction perpendicular to the direction of the vibration, may be described by the functional form $R(v_x) = \exp[-(|v_x|/\sigma_x)^{3/2}]$, where $\sigma_x = \langle v_x^2 \rangle^{1/2}$ [11,12]. However, these experiments were conducted with particles that are relatively elastic, and the velocity correlations were either not measured or claimed to be negligible. Particles that collide while rolling lose more energy in comparison to particles that collide in free space, due to frustration at impact. In earlier work [13], this effect led to strong deviations from a Gaussian, even in a dilute system, where the mean free path was of the order of the dimension of the container. However, dissipation at the boundary was significant and the distribution did not represent the bulk properties of the particles. These observations lead us to ask if there are correlations in dense systems where the mean free path is much smaller than the system size, and if these correlations affect the distribution of the velocities.

In this Rapid Communication, we address these questions by measuring the velocities of particles rolling on an inclined plane. This geometry slows the dynamics and allows us to vary the dissipation over a wide range. Energy is supplied to the system by means of an oscillating wall. We report the density and velocity distributions as a function of the number of particles N in a regime where the distributions are insensitive to the frequency or phase of the drive. The dissipation increases with N because the time between collisions as well as the mean free path decreases. Because the density of the particles becomes higher near the driving wall as N is increased, our analysis of the velocities is done in a narrow region where the density and the velocity fluctuations are relatively uniform.

We observe that significant correlations in the velocities of the particles are found that increase in strength and range as N increases. The distribution $P(v_x)$ of the velocity components of the particles perpendicular to the direction of the oscillation deviates strongly from a Gaussian. At low N, corresponding to lower dissipation, $P(v_x)$ may be fitted to $R(v_x)$ as in Refs. [11,12], but $P(v_x)$ deviates strongly from this form as N is increased. The kurtosis of $P(v_x)$ is observed to be as much as *four times* the value corresponding to a Gaussian distribution, signaling a significant breakdown of the assumptions of negligible velocity correlations in our system.

The experimental system consists of stainless-steel spheres with diameter $d = 0.3175$ cm rolling on a 29.5 cm $\times$ 27.0 cm glass plane that can be tilted from $0 = \theta \leq 8°$; a schematic of the experimental setup is given in Ref. [13]. The coefficient of rolling friction is observed to be less that 1×10^{-3}, and therefore energy loss during rolling is negligible compared to the loss that occurs due to a collision [14,15]. The number of particles is varied from $N = 100$–1000. To track the positions of the particles, we utilize a high-speed digital camera (Kodak Motioncorder) at a frame rate of 250 per second. A centroid technique is used to follow the particles at subpixel accuracy, allowing us to resolve positions to within 0.13 mm and velocities to within 5.2×10^{-4} cm s^{-1}. The statistical properties are obtained by averaging over time using at least 8500 frames.

Energy is added to the system by means of the bottom wall that is attached to a solenoid powered by a wave-form generator. The driving signal is not sinusoidal, but a periodic pulse with an amplitude $\mathcal{A} \approx 2.5d$ and a velocity $v_{\text{wall}} \approx 20$ cm s^{-1}. When the system is tilted past $\theta = 2.0°$ and the driving frequency $f \gtrsim 4.0$ Hz, the positions of the particles are observed to be independent of the phase of the drive.
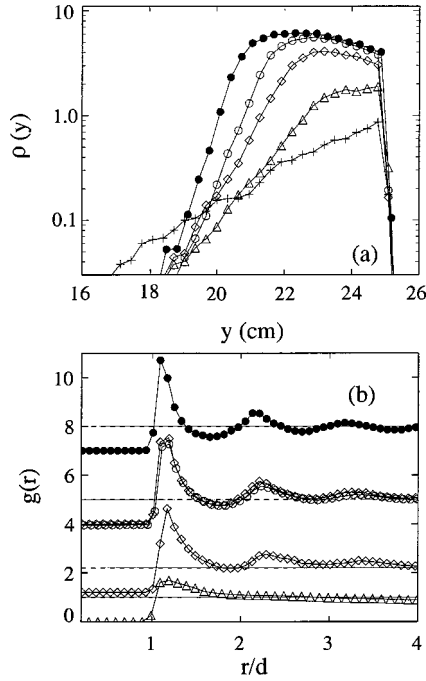
FIG. 1. (a) The average number density $\rho(y)$ in the y direction for various values of N; 1000 ($\bullet$), 750 ($\bigcirc$), 500 ($\diamond$), 200 ($\triangle$), and 100 ($+$). The driving wall is at $y = 26$ cm. (b) The radial distribution function $g(r)$ for the same values of N at $f = 6$ Hz (d is the particle diameter). The plots of $g(r)$ have been shifted for clarity, and the dashed lines show $g(r) = 1$. To show that the details of the driving do not influence the correlation of the particles, $g(r)$ for $N = 750$ at $f = 6$ Hz ($\bigcirc$) and $f = 20$ Hz ($\diamond$) are shown.

Below 4.0 Hz and for $N > 200$, we observe particles forming subharmonic patterns similar to those observed in vertical vibrated quasi-two-dimensional geometries [16]. The nature of the velocity fields and density distribution in this latter regime will be discussed elsewhere [17]. All of our results reported here are at a fixed $\theta = 3.0°$, $\mathcal{A} = 2.5\,d$, and $f = 6.0$ Hz. For these parameters phase dependence is absent in the density and velocity distributions.

The x and y axes are chosen to be along the driving wall and perpendicular to the driving wall, respectively. The origin corresponds to the point where the top wall intersects the side wall. The distribution of the particles in the x direction is uniform. The y dependence of the number density $\rho(y)$ is obtained by averaging the number of particles in bins of width d and dividing by the area of each bin [see Fig. 1(a)]. The density $\rho(y)$ is not constant because of gravity and the action of the driving wall, although for systems with $N > 100$, $\rho(y)$ shows a plateau where there are very small density gradients, similar to earlier observations [18,19]. Most of the particles remain suspended above the driving wall and receive random kicks from the particles that are between the bulk and the driving wall.

To ensure that the deviations in density over the compacted region do not influence the distribution of velocities, we varied the region over which the analysis of the velocities was performed. We found that the distribution of velocities is independent of the distance y in the compacted region. Therefore, we measured the velocity distributions in a region
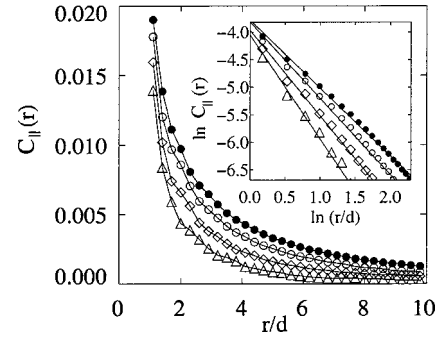

FIG. 2. The spatial correlation function of the velocities $C_\parallel(r)$ as a function of separation r/d, for $N = 1000$ ($\bullet$), 750 ($\bigcirc$), 500 ($\diamond$), and 200 ($\triangle$). Inset To show the nonexponential decay of $C_\parallel(r)$, we plot $\ln C_\parallel(r)$ vs $\ln(r/d)$. The solid lines are least-squares fits to the data. The slope $\alpha = -1.23$ ($\bullet$), -1.38 ($\bigcirc$), -1.55 ($\diamond$), and -1.90 ($\triangle$). The statistical error bars are smaller than the symbols.

of width $5d$ beginning $8d$ away from the driving wall.

To further characterize the compacted region in our experiment, we measured the radial distribution function $g(r)$ as a function of the distance r between the centers of the particles [7]. Our results for various N are shown in Fig. 1(b). The plots demonstrate a transition from a gaslike state for $N \lesssim 200$, to a more liquidlike state for $N \gtrsim 500$.

We now discuss the detailed properties of the velocities of the particles. The spatial correlation function of the velocities is given by

$$C_\parallel(r) = \sum_{i \neq j} \frac{\vec{v}_i \cdot \vec{v}_j}{|\vec{v}_i||\vec{v}_j|}, \tag{1}$$

where i and j label particles separated by a distance r. With this definition, two particles with parallel (antiparallel) velocities give a correlation of $+1$ (-1). We find that long-range velocity correlations are present for all N as shown in Fig. 2. We have plotted $\ln C_\parallel(r)$ versus $\ln(r/d)$ in the inset of Fig. 2. The r dependence is nonexponential, and least squares fits to $\ln C_\parallel(r)$ suggest a possible power-law decay. However, our range of r is limited by the finite size of our system. The values of the slope α are given in the caption of Fig. 2.

We now discuss how these correlations affect the distribution of velocities in the x and y directions (see Fig. 3). The velocity components v_x and v_y have been scaled by $\sigma_x = \langle v_x^2 \rangle^{1/2}$ and $\sigma_y = \langle v_y^2 \rangle^{1/2}$. The maximum of the distribution $P(v_x)$ is scaled to be unity for clarity and is given in Fig. 3(a) for various N. A Gaussian fit is shown for comparison. For low densities, we find that the form of $P(v_x)$ is similar to that observed in previous experiments with low dissipation [9,11]. For $N = 100$ ($+$), the form of $P(v_x)$ can be fitted by $R(v_x) = \exp[-(|v_x|/\sigma_x)^{3/2}]$, shown by the dashed line. However, as N is increased, the deviations are observed to become stronger and cannot be described by this power-law form.

The asymmetry of $P(v_y)$ seen in Fig. 3(b) occurs because the particles moving toward the piston have lost energy be-
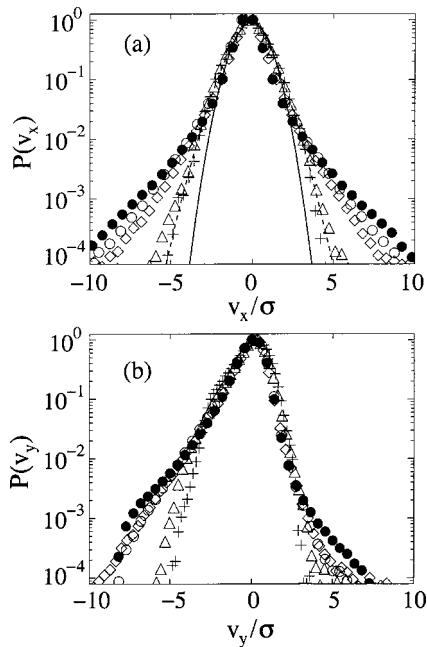
FIG. 3. The distribution of velocities for $N=1000$ ($\bullet$), 750 ($\bigcirc$), 500 ($\diamond$), 200 ($\triangle$), and 100 ($+$). (a) The velocity distribution $P(v_x)$ for the region described in the text. Each distribution is normalized to be unity at its maximum. The deviation from a Gaussian (solid line) is significant for all N and show a systematic increase as a function of N. In the most dilute case ($+$) $P(v_x)$ can be described by $R(v_x)=\exp[-(|v_x|/\sigma_x)^{3/2}]$, shown by the dashed line. (b) The distributions $P(v_y)$ are observed to be asymmetric due to the driving wall and dissipation. The asymmetry is observed to increase as N is increased. However, this asymmetry does not cause the deviations from a Gaussian in $P(v_x)$ (see text).

FIG. 4. (a) The kurtosis F_x of $P(v_x)$, as a function of N ($F=3$ for a Gaussian.) The solid line is a least-squares fit to the data. (b) The mean free path ℓ as function of the number of particles N. (c) The standard deviations σ_x ($\bigcirc$) and σ_y ($\triangle$) (measure of granular temperature) as a function of N, taken from the distributions in Fig. 3. The difference of σ_x and σ_y reiterates that the granular temperature is a nonscalar quantity. The line is a guide for the eye.

cause of collisions. The distributions become more asymmetric as N is increased because of the greater number of collisions between particles. To characterize $P(v_y)$, we calculate its skewness (third moment), and find that it ranges from 1.62–4.45 for $N=100$–1000, respectively. The effects of this asymmetry on $P(v_x)$ can be determined by discriminating particles with $v_y<0$. After performing this ''filtering,'' we find that $P(v_x)$ is insensitive to the sign of v_y. Thus the asymmetry of $P(v_y)$ is not the cause of the deviations in $P(v_x)$ from a Gaussian.

By using the kurtosis or flatness of the distribution $P(v_x)$ given by $F_x=\langle v_x^4\rangle/\langle v_x^2\rangle^2$, we can quantify the deviation of $P(v_x)$ from a Gaussian. If the distributions were Gaussian, $F_x=3.0$. If the distributions were given by $R(v_x)$, $F_x=3.762$. Figure 4(c) shows F_x as a function of N. The increase in F_x shown in Fig. 4(a) implies that $P(v_x)$ deviates more strongly from a Gaussian as dissipation is increased. The linear dependence of F_x on N may be due to the finite range of N.

These results show that there are strong correlations, both in the positions and in the velocities, as a consequence of the high dissipation that occurs due to collisions. If steel particles collide head-on in free space, they lose energy as determined by the coefficient of normal restitution given by $\eta\approx0.93$. However, when rolling particles collide, the translational velocity changes at the instant of collision, but the

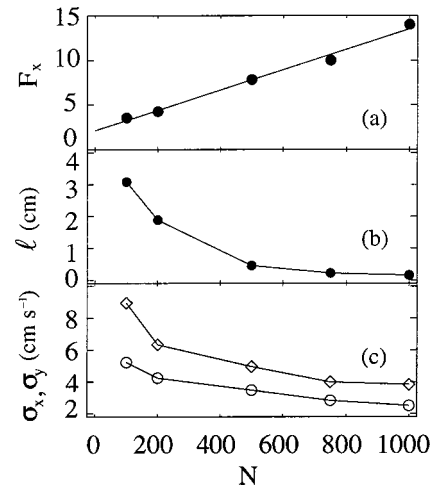angular velocity does not. Therefore, the rolling condition is no longer satisfied and the particles slide for a short duration. Because the sliding friction is about 100 times larger than the rolling friction for steel on glass, the rolling condition is established within a few particle diameters [13–15]. Hence, particles lose more energy than they would by considering only the normal coefficient of restitution. Measurements for our system show that the effective coefficient of restitution $\eta_{\rm eff}\approx0.5$ [13]. Because the effect of inelasticity is to reduce relative motion [20], this smaller value implies that the particles tend to cluster and stream when driven, leading to the observed spatial and velocity correlations.

To estimate the increase in dissipation with N, we calculate the mean free path ℓ from the relation given in Ref. [21]. The results for ℓ as a function of N are shown in Fig. 4(b). We find that ℓ decreases much faster than the estimate based on a uniform spatial distribution of particles. By dividing ℓ by the root-mean-square velocity, we obtain the mean collision time between particles. This time is observed to decrease from 0.83 s to 0.059 s as N is increased. The increase in the range of the velocity correlations and the deviations from a Gaussian in $P(v_x)$ is due to the higher dissipation in the system for larger N. As N is increased, the collision rate increases, which gives rise to higher dissipation. As the latter is increased, the overall particle velocities decrease. This decrease in turn leads to more compaction and smaller mean free paths.

To ensure that the means of energy input is not effecting the system, we have varied the driving parameters. We find that the statistical properties do not depend strongly on how the system is driven above a threshold $f\gtrsim4$ Hz. Increasing f and the amplitude $\mathcal{A}$ of the oscillating wall by a factor of five has no effect on the spatial and velocity distributions for fixed N. For example, Fig. 1(b) shows that for $N=750$, $g(r)$ remains unchanged if f is increased by a factor of 3.3 and $\mathcal{A}$

is increased by a factor of 3.0.

In Fig. 4(c) we show σ_x and σ_y as a function of N to confirm the anisotropy of the granular temperature for our system. The plot also shows that the average energy per particle decreases as N is increased. This decrease occurs due to the higher collision rate between the particles that removes energy from the system through dissipation.

In summary, strong velocity correlations in dense granular gases have been experimentally reported. These correlations are observed to cause the velocity distributions to deviate significantly from a Gaussian. Our experiments, in addition to previous reports [10], reconfirm that the velocity distribu-tions depend on the degree of dissipation in the system and that the form $R(v_x) = \exp[-(|v_x|/\sigma_x)^{3/2}]$ of the velocity dis-tribution is not universal.

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